

RESEARCH PROPOSAL

RHODES UNIVERSITY, EDUCATION DEPARTMENT (Mathematics Education)

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Degree: PhD

Provisional Title

A critical analysis of how the potential of Dynamic Geometry Software as a visualisation tool may enhance the teaching of Mathematics.

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Abstract

GeoGebra, an open source dynamic geometry software package, is a powerful visualisation tool for the exploration of mathematical concepts, properties and ideas. This research study is conceptualised within the GeoGebra Literacy Initiative Project (GLIP) – an ICT teacher development project in Mthatha in the Eastern Cape. The objectives of GLIP is to develop the appropriate ICT skills to use GeoGebra effectively and strategically as both a teaching and learning tool of mathematics. The focus of this study is on how GeoGebra could be used as a teaching tool in harnessing its powerful visualisation capacity. In the study, selected GLIP teachers collaboratively develop GeoGebra applets, implement them and evaluate them. As visualisation plays a vital role in developing mathematical ideas the focus in the design and implementation of selected applets will be on their visual appeal and capacity to develop and grow teaching proficiency.

The empirical component of this study centres on an intervention programme that capitalises on the community of practice that has developed in the GLIP. The research methodology takes the form of action research cycles in which the design, implementation and evaluation of successive applets determines the data gathering and analysis process. The underlying theoretical foundation of this study lies in constructivism which aligns well with the conceptual and analytical framework of Kilpatrick's et al. (2001) teaching proficiency. As the participating teachers implement their applets in a teaching environment, their teaching practice will be analysed in terms of the applets' visualisation power using aspects of teaching proficiency such as teaching for conceptual understanding and teaching fluency.

Keywords: Teaching with dynamic geometry software, GeoGebra, visualisation, teaching proficiency.

Common Statement

This PhD study, with a focus on **teaching**, is paired with another PhD study, also within the GLIP, that focuses on aspects of **learning** with applets in the context of using visualisation in mathematics education.

PROVISIONAL TITLE

A critical analysis of how the potential of Dynamic Geometry Software as a visualisation tool may enhance the teaching of Mathematics.

1 Introduction

The ever-growing use of technology in all realms of life has also affected teaching styles in many parts of the globe. Research shows that ICT can influence teaching mathematics positively and has great potential for conceptually enabling many children to see and access a variety of mathematical ideas (Jones, 2000; Keong, Horani, & Daniel, 2005).

The purpose of this research study is to:

- analyse how teachers make use of ICT technology-aided visualisation for effective teaching,
- analyse the advantages and weaknesses of using Dynamic Geometry Software in teaching and learning of mathematics,
- to interpret, through elements of an action research approach, the pedagogical practices and instructional fluencies when visual aspects of technological tools are employed in a constructivist classroom,
- contribute to the growth of a community of proactive teachers in Mthatha collaborating in building IT resources (applets) aligned with the South African curriculum content within the context of GLIP.

Alsina & Nelsen (2006, p. 121) argue that the interactive possibilities of **technology** may foster rich constructions of mathematical concepts and develop visual thinking. Dynamic Geometric Software is a technological tool which provide an opportunity for discovery and exploration, thus opening new possibilities for visual experiences in mathematics teaching (and learning). Alsina and Nelsen (2006) also suggest that **visualisation** in the classroom has its own pedagogical values. In the heuristic of mathematical discovery, internal visualisation (such as imagination and drawing pictures in the mind) and external visualisation (such as making use of sketches and drawings) plays a major role in developing intuition and problem solving skills. This study analyses how dynamic software can enhance teaching by exploiting the power of visualisation in a constructivist classroom. The study also examines critical aspects of this software that limits the usage of visualisation in the mathematics classroom.

2 Context of the study

The Department of Basic Education (2011) has identified mathematics as a priority subject as, 'it helps to develop mental processes that enhance logical and critical thinking'. Mathematical problem solving also 'enables us to understand the world around us'. An analysis of the National Senior Certificate (NSC) November 2014 and 2015 Grade 12 results indicates that the performance in mathematics in Eastern Cape has dropped from 42% to 37.3% and is the lowest performing province in South Africa (Department of Basic Education, 2015). The Department of Basic Education has implemented ICT based teaching and learning as part of the National Strategy of MST (Mathematics, Science and Technology). The Ministerial Committee (2013, p. 11) however reports that the Eastern Cape Province faces a serious shortage of qualified and proficient teachers who should be implementing this strategy,

particularly in the mathematics classroom. Further, Ministerial Committee (2013) suggest that the ICT resources in schools are not adequate and where available, they are not used to their full potential. It is thus of particular significance that this study speaks to this policy. This study, with its intervention, will hopefully provide significant input into how interesting ICT resources can be harnessed for proficient teaching in mathematics.

GLIP – GeoGebra Literacy Initiative Project

GLIP is a teacher development project designed specifically for interested teachers to use GeoGebra, a dynamic geometry software, as a teaching and learning tool of mathematics. GLIP trains teachers to not only use GeoGebra, but also to harness its interactive capacity to develop tailor made applets for their teaching. Applets are small programs written in Java that can be easily embedded in web pages. This allows teachers and learners to access them at any time and in any place when connected to the internet. The project was launched in November 2015 with the participation of 12 mathematics teachers from one school in Mthatha, Eastern Cape. It is envisaged that GLIP will involve other schools in the region in 2016/2017. There are two phases in the program – Phase 1 involves the training of teachers and learners; Phase 2 will be the use of the GeoGebra in classrooms. The first phase of GLIP consisted of introductory training in GeoGebra. This helped teachers to familiarise themselves with the software. During this phase, teachers were trained and encouraged to explore and play with different options available in GeoGebra to realise and appreciate its potential in mathematics classrooms. This first phase was spread across 2 weeks and consisted of 4 sessions of 12 hours each. In Phase 1 teachers were introduced to existing applets and were encouraged to use them and reflect on how these could be used in their teaching. After this the teachers were trained how to develop their own applets. The applets were then implemented and piloted in the classroom for student exploration. The materials for the first phase of this workshop were adapted from those of Hohenwarter’s (2009), downloaded from the website on the 26th August 2015 with his consent to adapt and use for our training purposes. Currently, the training of learners in GeoGebra is in progress. After that GeoGebra will be implemented in the classrooms (second phase of GLIP) as per the intervention programme of this study, refer Table 1 below.

Cycles	Topics for Applets
1 applet: 1 X 2 sessions	Circle Geometry Cyclic Quadrilateral;
2 applets: 1 X 2 sessions	Functions Vertical and Horizontal Shift Relation Turning Point and Intercept
2 applets: 1 X 2 sessions	Algebra Nature of roots Linear and quadratic inequalities
1 applet: 1 X 2 sessions	Finance Simple and Compound Decay
1 applet: 1 X 2 sessions	Analytical Geometry Slope of line with tangents
1 applet: 1 X 2 sessions	Trigonometry Sine rule

Table 1 GLIP Structure

This study is located in the second phase of GLIP, where the participating teachers will start developing and using GeoGebra applets in their classrooms. The development of these applets will determine the sequence of the GLIP cycles. A GLIP cycle commences with a selection of a topic or sub-topic, which is determined by the annual teaching plan. The next step of the cycle is the planning of pedagogical strategies to be considered while designing the applet. The teachers brainstorm on how to teach using technology. One or two teachers will develop the applet and open up for a discussion on the applet and it will be modified whenever required. Once applet is developed, it will be used by all the participant teachers in the classrooms. After teaching with applets, the teachers will again come together to reflect and evaluate their lessons, which will inform the development of next cycle of applet. We will generate 8 applets involving at least 2 group discussions per applet. The anticipated structure of GLIP in 2016 is as per Table 1 above.

The flow of development and deployment of applets is discussed further under data collection. The GLIP is driven by me and my co-researcher. We also were involved in training the teachers in GeoGebra skills. After the training, we assumed the role of participants in GLIP. Through this project, the teachers will be empowered to use technology in teaching and learning of mathematics. This research study will run parallel to the second phase of GLIP.

3 Literature Review

3.1 Conceptual Landscape – Visualisation

This study focuses on the role of visualisation in teaching and learning of mathematics through ICT, in particular the use of GeoGebra. A Chinese proverb says "One picture is worth ten thousand words". Arcavi (Arcavi, 2003) emphasises that the visual representation of information in a graph, for example enables us to comprehend a scenario or relationship between two processes without reading sequentially and logically printed words. He also sees mathematics as a subject which represents real world scenarios many of which appear to be visual. Mathematics makes use of a variety of visual imagery. Presmeg (Presmeg, 1986b) defined visual imagery as a mental schema in the presence or absence of an object.

Lean & Clements (Lean & Clements, 1981b) following Hebb (1972) define visual imagery as 'the occurrence of mental activity corresponding to the perception of an object, but when the object is not present to the sense organ'. They emphasise the significance of visual imagery in teaching and learning of mathematics by stating that "many highly original and significant creations of the human mind have been largely the result of nonverbal mental representations (mainly visual imagery)." According to Lean & Clements (1981b), mathematical ability generally comprises of general intelligence, visualisation and spatial ability to generate and formulate mental images. They distinguished the ability to form 'memory' images and the ability to form 'abstract' images and if both these abilities exist, then the flexibility to switch between them (visual vs abstract), could be an important factor in solving mathematical problems.

Diagrams frequently accompany mathematical thinking (Krutetskii, 1976). In order to solve mathematical problems, one should perceive and use clear mental pictures. The ability to visualise abstract mathematical ideas help to solve complicated problems quickly and accurately. Krutetskii (1976) however found in his study of Soviet school children, that "mathematically able students have

no need for visualising objects or patterns even when the mathematical relation suggests visual concept”.

Guzmán (Guzmán, 2002) however considers visualisation to be a powerful communication and learning tool. He argues that mathematical activities like analysing a problem, problem-solving, demonstration of a problem as well as handling theorems involve some form of visual activity. He further argues that “mathematical concepts, ideas, methods, have a great richness of visual relationships that are intuitively representable in a variety of ways”. He emphasises that using different visual images to analyse, differentiate and manipulate concepts, enables one to solve problems in a versatile manner. Visualisation is therefore useful in teaching and learning mathematics. The basic mathematical ideas are born from concrete and visualisable situations. Visualisation is an important aspect in mathematical activity, through which one can explore different structures of concrete reality. The initial perceptions of similarities in the real objects or situations can guide learners to abstraction and symbolic representation. Thus for Guzmán (2002) visualisation in mathematics is the ability to relate and handle both concrete and corresponding abstract objects. For the purpose of this study the following definition of visualisation by Arcavi (Arcavi, 2003) is adopted:

Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings.

Arcavi (Arcavi, 2003) suggests that visualisation offers a method of seeing the unseen. For example, a graphical representation of tabular data helps us to see and understand some broad features of the data. The graph assists us in visualising a particular mathematical relationship. It is important however to understand that visualisation as a process does not exclude the process of verbalisation. To the contrary, the two processes may well complement each other, although the focus of this study is on visualisation.

Duval (Duval, 1999) observes that visual perception is complex and at times perplexing. Although we live in a three dimensional world, many diagrammatic representations and images appear to be two dimensional i.e. we can only see one side of an object. For the complete apprehension of the object the observer requires to ‘move around’ the object and visualise the other sides of the object. In my experience, many of my learners find this type of visualisation difficult. Visual perception needs exploration through physical movements whereas visualisation can get a complete comprehension of the object at once. According to Duval “understanding involves grasping the whole structure and there is no understanding without visualisation”. Duval (Duval, 2013) states that visualisation is the ability to discriminate what is irrelevant and what is relevant in a given context. For example, in the Figure 1.1 AE, AF and BC are tangents to a circle. Is the perimeter of the triangle ABC equal, greater, smaller than the sum of the line segments AE and AF? (adapted from Duval (1999)).

In the first instance, a student may find it difficult to answer since the lengths are not given. But the moment the sub-configuration of BC is considered as two segments, $BC = BD + DC$, it is easy to infer that BD and BE are congruent, by virtue of the behaviour of two tangents from a common point. Visualisation lies in discarding the irrelevant segments AB, AC and focusing on the line segments BE and BD, refer to Figure 1.2. Thus $BC = BE + CF$, and so the perimeter of triangle ABC will be equal to

the sum of AE and AF. Teachers thus need to organise appropriate learning sequences to help learners, “to embrace the whole range of variations of the conditions of a problem and to bring out the various factors that make them clear” (Duval, 1999).

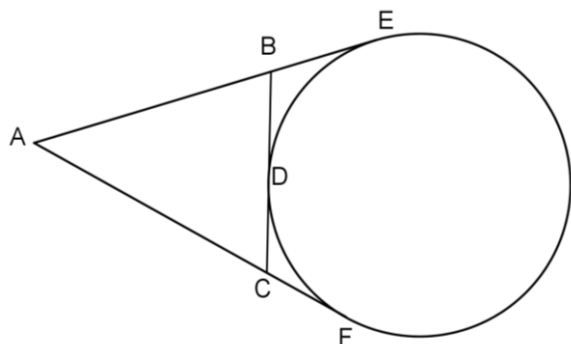


Figure 1.1

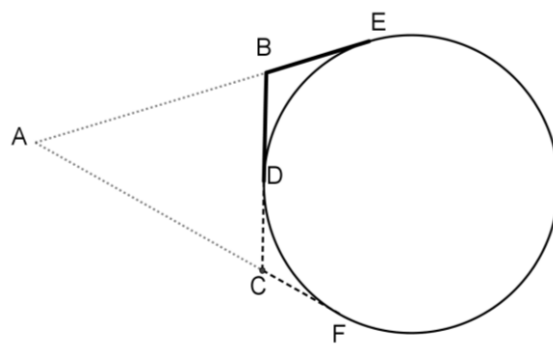


Figure 1.2

Some topics in higher secondary mathematics need greater visualisation abilities than other topics. Consider the following function $f(x) = x^2 - 4x + 3$ having the turning points at $(2, -1)$. If asked to find the turning point of $f(x + 3) - 2$, students typically find a new equation and recalculate the turning points algebraically. However having a clear mental image of the parabola can assist to simply translate the turning point by 3 units left and 2 units down resulting in the point $(-1, -3)$. Although the question had no mention of graphs, strategic use of visualisation can simplify the problem and yield appropriate solutions. Research and my own experience shows that the following Grade 11 topics are found to be difficult for students with poor visualisation practices:

- Sketching of graphs; (Lean & Clements, 1981b)
- Interpreting graphs (Find the equation of a given graph, for which value of x is $f(x) > 0$, for which value of x is $f(x).g(x) < 0$)
- Translation and reflection of graphs;
- Interpreting two dimensional impacts of three dimensional trigonometry/geometry situations (Angle between two planes) (Lean & Clements, 1981b)
- Interpreting dimensions of three dimensional geometric solids (to distinguish between height (h) and slant height (s)).

Presmeg (1986a) defined mathematical visualisation as “the extent to which a person prefers to use visual methods when attempting mathematical problems which may be solved by both visual and non-visual methods”. She classified individuals with regard to mathematical ‘visuality’ into three groups namely non-visualisers, visualisers and a middle group. It is thus important for teachers to choose carefully their teaching strategies to suit these individual differences. Invariably teachers face different types (with respect to visuality) of learners in their classes. Presmeg (Presmeg, 2014) reveals that visualisers in a class of a nonvisual teacher are like fishes out of water, and their learning is compromised. Teachers need to bring in different teaching styles into their practice to accommodate all learners. I concur with Presmeg (Presmeg, 1986a, 2014) that when teachers prefer non-visual mode of teaching, visual learners encounter difficulties in abstracting and generalising of mathematical ideas. Hence teaching and learning is not always effective. When we make connections between mathematics topics and the real world in a classroom, we should employ visualisation for effective learning. Presmeg (2014) also argues that visual thinking is fruitful in promoting creativity. Thus there is a need for

teachers to take advantage of visualisation for effective teaching and learning of mathematics. Lean & Clements (1981a) also raise the concern, 'which is the best form of instruction for a person who prefers a visual mode of response (or, similarly, a verbal-logical mode)?'

Arcavi (2003) asserts that technology may assist in developing visualisation skills so that students are able to better 'see' mathematical concepts and ideas, helping to overcome the limitation of visual perception. Thus visualisation sharpens our understanding. By hearing the description of a concept our imagination will create an image which we attach with the description. I argue in this study that seeing these concepts with the help of ICT technology, for example could sharpen our imagination and subsequent enjoyment and better understanding of the concept. I will argue in the next section that ICT technologies can be used effectively to enhance the appreciation of mathematical ideas and concepts through its inherent visualisation capacity.

3.2 Technology – Dynamic Geometric Software

"Technology potentially opens up new observation possibilities for teachers, allowing them to focus on students' investigations and thinking strategies while solving mathematical problems"(NCTM, 2000, p. 25). Research indicates that despite the numerous benefits of using ICT in mathematics education, the process of embedding ICT in classrooms is a slow and complex process (Hohenwarter & Lavicza, 2007). From my observations, many of today's teachers and students have access to computers, laptops and tablets. However, technology is mainly used by teachers for administrative purposes and monitoring of students' assessment but not often for effective teaching and learning. "For the majority of teachers, solely providing technology is insufficient for the successful integration of technology into their teaching" (Cuban, Kirkpatrick, & Peck, 2001). However, recent research published at MIT by Carter, Greenberg, & Walker (2016) found in their randomised experiment that computer devices can have a substantial negative effect on academic performance. This negative effect may be due to the ineffective use of this technology by the teacher.

Stols et al. (2015) argue from their own observations that as a result of teachers' own perceptions that their skills and knowledge are limited, they often refrain from using technology in teaching and learning of mathematics. In particular, Stols et al. find that the use of GeoGebra in South African classrooms is very limited. Nevertheless, teachers carry a positive sentiment that the technology-supported teaching is very effective and it will promote visualisation and student understanding of mathematics. Being aware of the vital role of technology in classrooms, appropriate professional development programs and easily accessible and modifiable teaching materials are essential, so as to integrate technology into teaching practices. Through training programmes, like GLIP, teachers can thus improve their teaching methods and broaden their instructional repertoires.

Arcavi (Arcavi, 2003) asserts that there is a need for cognitive technologies that help transcend the limitations of the mind. Such 'technologies' are those that use visual tools to better 'visualise' mathematical concepts and ideas. Ruthven et al. (Ruthven, Hennessy, & Deaney, 2008) argue that Dynamic Geometry Software (DGS), helps to produce accurate figures and images including their measurements such as angle and length. DGS is particularly suited to make use of other facilities such as dragging, sliding points and lines on the computer screen. Hoyles & Noss (2003) affirm that teachers should use DGS as a pedagogic tool for the exploration of mathematical concepts and ideas for any mathematical domain. Key to effective DGS is an interface that affords direct manipulation of artefacts

that can be dragged around the computer screen but keeping its constructed properties and underlying relationships preserved. Figure 2 below, a typical applet, is an example of a dynamic GeoGebra visualisation that allows teachers to guide students to discover the relationship between the angle at the centre of a circle and the angle at the circumference of the circle subtended by the same arc BC. The learner can drag the slider α to change the size of angle $B\hat{O}C$ and observe what happens to angle BDC, hence verifying its relationship with the angle BOC. By engaging with this activity, learners may discover that in spite of changing the magnitude of the angles, the relationship between $B\hat{O}C$ and $B\hat{D}C$ is preserved i.e. that the angle at the centre of a circle is always twice the angle at the circumference of the circle. The above applet can be accessed online at <http://ggbm.at/t2Qf2dDV>. Proponents of DGS also suggest that it has the potential to promote a discovery method of teaching and learning. One of the important features of DGS is that it provides a visual interface to create interactive applets, as explained above, which can be integrated into the mathematics classroom in the form of virtual manipulatives.

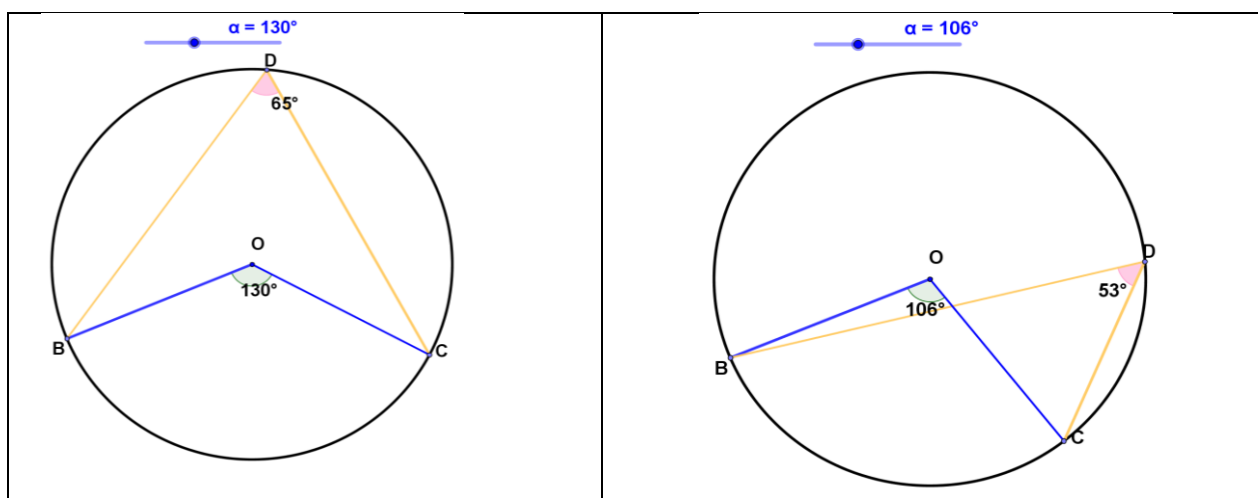


Figure 2

Moyer et.al (2002) defines virtual manipulatives as a web-based, visual, dynamic interactive environment which helps to construct mathematical knowledge. Virtual manipulatives take the form of 'applets', 'mathlets', (Durmuş & Karakirik, 2006) or 'dynamic worksheets'. The mere presence of dynamic software does however not guarantee the acquisition of knowledge. Moyer (Moyer, 2001) argues that teachers play an important role in creating interactive mathematics environments that provide students with representations that enhance thinking. Teachers need to be skilled in working with virtual manipulatives with interactive capabilities to facilitate opportunities for constructing knowledge in their classrooms. Rivera (2011) confirms that the conceptual content of the visual manipulatives matters as individuals need to establish a valid and consistent mapping between the visual representation and the corresponding concept. In mathematics, these manipulatives can be used in a wide variety of topics.

Durmuş & Karakirik (2006) argue that the use of virtual manipulatives not only increase students' conceptual understanding and problem solving skills but also promote their positive attitudes towards mathematics. Manipulatives can provide "concrete experiences" that focus attention and increase motivation. Virtual manipulatives can provide an interactive environment for solving problems by making connections between mathematics concepts, operations and the real world. Teacher demonstration of manipulatives alone is however insufficient for conceptual understanding. Students

need to be given the opportunity to work with these manipulatives and discover the mathematical concepts and relations themselves. Virtual manipulatives are particularly well suited for an interactive environment where students can pause, reflect, experiment and solve problems to make connections between mathematical concepts because the devices on which the manipulatives are operated can be stopped at any time. It cannot be overemphasised that every learner should get an opportunity to engage with manipulatives. By doing so, they enrich their understanding of concepts and relations. Giving credit to Norman (1993), Durmuş & Karakirik (2006) argue that computers "help us not only to make sense out of what we have experienced and what we know but also to compose new knowledge by adding new representations, modifying old ones, and comparing the two."

There are many DGS packages available for teaching and learning such as Geometer's Sketchpad, Cabri and GeoGebra (Bu, Mumba, Henson, Wright, & Alghazo, 2010; Ruthven et al., 2008). This study focuses on GeoGebra as a dynamic geometric software for two reasons. Firstly, GeoGebra is an open-source dynamic mathematics software which means it is available free of charge without any licensing issues. This allows teachers and learners to use it in any way they wish. Secondly, it is a platform independent software which means that it runs virtually on any operating system as it requires only a Java plug-in. Although this software was developed in the early 21st century to incorporate geometry, algebra and calculus, it has now been fully developed incorporating other mathematical perspectives such as statistics, probability and also 3D graphics. GeoGebra has rapidly gained popularity among teachers and researchers around the world, as it is easy to use and combines many aspects of different mathematical ideas (Hohenwarter & Lavicza, 2007). It provides a resourceful dynamic learning environment for mathematics teachers to integrate mathematical content and pedagogical strategies for the purpose of teaching mathematics for understanding (Bu, Mumba, Wright, & Henson, 2012, p. 91). It also offers a powerful opportunity for teachers to create interactive learning environments.

Bu et al. (2012) observe that participants of the GeoGebra community not only actively invent and experiment with new ways of teaching mathematics, but are themselves learning or relearning mathematics through their applets and manipulatives. The GeoGebra community has a large number of international users and developers, from almost 200 countries. Currently in excess of 253 635 GeoGebra applets are available online which can be accessed across the world by teachers, learners and researchers alike ("GeoGebra," n.d.).

Tasks in GeoGebra are useful when learners are encouraged to uncover the hidden relationships (Holzl & Schafer, 2013). The interactive nature of dynamic applets has the potential to promote student's understanding, which would have been difficult in a static environment. With the help of GeoGebra, teachers can ask open-ended questions like what-if and what-if-not, thereby supporting and guiding students to discover properties themselves. In GeoGebra there are tools available that enable users to construct objects in many different ways. For example, the 'parallel line' tool is used to construct parallel lines; the polygon tool is used to construct polygon etc. GeoGebra allows the flexibility to enable or disable certain tools in a dynamic worksheet. On one end we can create an applet with no tools available. On the other end, students can be given fairly open ended problems with tools to construct and solve them. Teachers need to consider the pedagogical implications and learning opportunities available when setting tasks using GeoGebra. Sherman (2010) suggests that students' prior mathematical knowledge and their skill with GeoGebra must also be looked into when such tasks are given to students.

The functional use of technology is to carry out complex and routine procedures for solving problems. In mathematics classrooms, Kendal & Stacey (2001) maintain that technology can be used 'functionally' and 'pedagogically'. The pedagogical use is to develop conceptual understanding of mathematical ideas. Examining different teaching approaches using technology, Kendal & Stacey (2001) observe teachers typically make their own pedagogical choices about how to incorporate technology into their classrooms. Some teachers embrace technology as it enables them to increase their repertoire of different ways of teaching rules and procedures for student academic achievement, while some teachers find the capabilities of technology superior for developing conceptual understanding of mathematical ideas.

3.3 Teaching Proficiency

In most countries of the world teachers teach across numerous grades. Schoenfeld (2008) argues that this practice provides teachers with a sense of curricular continuity and mathematical depth which shapes their current instructions. By adopting the above practice, a deeper understanding of the core mathematical ideas is evolved. Schoenfeld (2008) makes the observation that the more advanced teacher's mathematical knowledge is, the better the teacher can make connections between mathematical topics. He suggests that the characteristics of proficient teachers are:

- having a broad and connected mathematical content knowledge
- being aware of learner's prior knowledge
- being aware of the entire curriculum sequence
- being able to effectively introduce a new mathematical idea
- being able to develop understanding in multiple ways
- who help to work with misunderstanding to develop correct understanding?

Proficient teacher's knowledge of school mathematics is broad. It is broad in the sense that proficient teachers can represent concepts in multiple ways and make appropriate and rich connections to other topics. A teacher needs to know the curriculum and have a sense of future directions of the mathematical content and what mathematical knowledge has been taught. This awareness allows teachers to plan for effective ways to introduce a new mathematical concept so that it fits with the existing knowledge. Hennessey, Higley, & Chesnut (2012) emphasise that links to prior knowledge and multiple representation in teaching mathematics not only strengthens learning experiences but also increases retention and application. Teachers should be aware of common misconceptions and how to deal with them.

Proficiency in teaching is related to effectiveness, consistently helping students to learn worthwhile mathematical content. Kilpatrick, Swafford, & Findell (Kilpatrick, Swafford, & Findell, 2001) propose five interwoven strands of teaching for mathematical proficiency. They argued that effective teaching practice can be developed and nurtured (p. 369). In the context of proficient teaching the five strands are:

- Conceptual understanding of the core knowledge required in the practice of teaching;
- *Fluency* in carrying out basic instructional routines;
- *Strategic competence* in planning effective instruction and solving problems that arise during instruction;

- *Adaptive reasoning* in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them; and
- *Productive disposition* toward mathematics, teaching, learning, and the improvement of practice. (p380)

Due to the limited scope of this study, teaching proficiency with respect to using DGS will only be analysed against the first two strands articulated above.

1. Conceptual understanding of core knowledge

Kilpatrick et al. proposes that three kinds of knowledge are crucial for teaching school mathematics: Knowledge of mathematics, knowledge of students and knowledge of instructional practice. Knowledge of mathematics includes knowledge of mathematical facts, concepts, procedures, and the relationships among them. A teacher must be aware that mathematical ideas can be represented in multiple ways. Teachers' mathematical knowledge and their capacity to use it in teaching is crucial in developing student's mathematical proficiency. Teachers need to know how students think and be aware of their conceptions and misconceptions. One of the common mistakes that students make in algebra for example is that $(a + b)^2 = a^2 + b^2$. The teacher needs to identify such common errors and diagnose reasons as to why this mistake is so prevalent. The teacher then needs to develop a set of responses for helping students in removing such misconceptions. An example of an appropriate response would be to use visualisation processes to represent $(a + b)(a + b)$ as the area of a rectangle. Knowledge must be connected so that it can be used intelligently. This integrated knowledge of mathematics has implications for teaching. A proficient teacher needs to make use of all the resources, such as GeoGebra, that can facilitate this integrated knowledge to develop students' mathematical proficiency.

For Shulman (1987) the knowledge base of a teacher represents "the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented and adapted to the diverse interest and abilities of learners, and presented for instruction". Effective teaching takes place when a teacher is able to transform his or her content knowledge into teaching strategies that are pedagogically powerful. Here the teacher draws upon a variety of instructional approaches like examples, non-examples, manipulatives, simulations, or inquiry method that can lead to educational outcomes identified by the teacher. An important role of a teacher is to probe and provoke student creativity. Teaching proceeds through a series of activities during which students are provided opportunities for learning. Shulman (1987) argues that the knowledge base lies in the capacity of a teacher to understand student representations and to respond to those student ideas. Thus proficient teaching involves thinking and identifying multiple ways of representing 'concepts' to students.

2. Instructional fluency

Instructional fluency refers to knowledge and skills acquired by a teacher and knowing when and how to use procedures accurately, efficiently and appropriately in a given situation. A teacher having acquired a repertoire of teaching approaches can readily draw upon them as they interact with learners in teaching mathematics. Proficient teachers have a clear vision of the goals of instructions.

They need to be able to use their knowledge flexibly in practice to appraise and adapt instructional materials, to represent the content in honest and accessible ways, to plan and conduct instruction, and to assess what students are learning. (p.369)

These routines help in developing mathematical activities like how to respond to a learner with serious misconceptions or to deal with learners who lack basic skills. Since teachers have access to several approaches to teaching, if one does not work they should be able to switch to another approach. According to Ball & Bass (2000), fluency in teaching mathematics is the ability of a teacher to 'deconstruct mathematical knowledge where elemental components are accessible and visible.' Though abstraction is central in mathematics, it may obscure the roots of the knowledge. Ball & Bass (2000) observe that a proficient teacher must be able to work backwards from abstract ideas and unpack their constituent elements. Thus proficiency in teaching involves a special capacity to understand and appreciate student's insight and deconstruct highly abstract knowledge, making hidden elements visible.

Ma (2010) accentuates that a teacher with profound understanding of mathematics is not only aware of the conceptual structure and basic attitudes of mathematics, but is also able to teach these to students. Teachers with weak conceptual knowledge of mathematics tend to develop only the procedural fluency of learners. Teaching is a complex activity with interrelated components (Kilpatrick et al., 2001) and visualisation is crucial in developing mathematical ideas. Teachers should be able to integrate dynamic mathematics software into their practice thereby enabling learners to construct and visualise mathematical ideas. Bu et al. (2010) asserts that engaging students in cognitive tasks with GeoGebra applets for example, reinforces the importance of visualisation of mathematical concepts.

Visualisation and stimulation are the primary advantages of using dynamic software that supports the teaching and learning of abstract mathematics concepts (Naidoo & Govender, 2014). Teachers can play an important and active role in analysing, designing and developing applets to be used by the learners effectively. Naidoo & Govender (2014) assert that the teachers' involvement in observing learners' engagement with applets is essential whether or not they are enthusiastic and critical in their use of tools. Thus GLIP is an important platform and forum that offers an opportunity for teachers to plan their technological lessons focussing on how technological-based tools can be implemented in classrooms that positively promotes the teaching and learning process of mathematics.

4 Research Goals

The overall goal of this research is to investigate whether and how dynamic software aided visualisation can enhance the teaching of Grade 11 mathematics.

In pursuance of this goal, the following specific research questions will guide the study:

- How can Dynamic Geometry Software such as GeoGebra be used as a visualisation tool to teach Grade 11 Mathematics?
- What enabling and constraining factors do Grade 11 teachers encounter when using GeoGebra as a visualisation tool to teach Grade 11 Mathematics?

Even though GeoGebra is an open-source software and available free of cost, there is little evidence in the literature that GeoGebra is being used by teachers in their routine teaching of mathematics. Further

there is little empirical research in South Africa, especially in Eastern Cape of its efficiency in teaching secondary school mathematics.

This action research study involves 'teachers working together to improve educational practices' (Mertler, 2012) when GeoGebra is used in classrooms. The significance of the study is to empower teachers with the dynamic mathematics software GeoGebra and enable them to incorporate its powerful visualisation capacity in their teaching of mathematics. In the GLIP, available applets may be modified or new applets may be generated for teaching the specific mathematical content. These applets (artefacts) will be created in collaboration with all the participant teachers in GLIP. The teachers in GLIP will brainstorm on new teaching techniques using applets. During the implementation stage in the GLIP, my fellow researcher will focus his research on learners' interaction with these GeoGebra applets whereas I will explore how teachers use GeoGebra as a visualisation tool in enhancing teaching proficiency. In the process of this study, a community of proactive teachers will be formed, creating and sharing applets relevant to the South African context. This community may substantially contribute GeoGebra applets to the existing pool of online resources. I also anticipate that the teaching strategies using applets generated through these action research cycles, comprising of implementation and reflection of instructional methods using ICT, will enrich the community of teaching practice. This research will be a contribution to teacher training and teacher development programs through effective integration of ICT into the mathematics classroom. Further, the curriculum designers and policy makers will also be informed of the relevance of incorporating ICT as a pedagogical tool, in particular GeoGebra, into teacher in-service training courses.

5 Theoretical framework

Boaler (2009) argues that a traditional approach to teaching is characterised by chalk and talk methods where the teacher typically explains and demonstrates the mathematical content of a lesson and the students simply watch, consume and practice the problems, mostly in silence. However, Jaworski (Jaworski, 1994) argues that this method of direct instruction develops only 'lower level mathematical skills' like computation, but deep conceptual understanding requires 'higher order skills'. The key idea of constructivism is that children construct their own knowledge. Jaworski (1994) asserts that learners do not only absorb ideas presented by their teacher but rather that they create their own knowledge. We ourselves construct knowledge from our own experiences. Thus each individual's knowledge is unique and contextually dependent. Habib (2012) argues that substituting traditional teaching methods with active approaches, which is inherent in a constructivist approach, exploits the intuitive and applied activities related to life. This active approach aligns well with teaching with ICT technologies as it requires hands-on teaching and learning strategies, where the teacher and learner are actively engaged in constructing knowledge. This approach underpins both the intervention of this study and the assumptions about effective pedagogy.

Dynamic mathematics software is a powerful teaching and learning medium and appears to create opportunities for creative thinking (Stols & Kriek, 2011). Stols (2007) further argues that the new technologies can help learners to visualise difficult-to-understand concepts and help teachers to create an active learning environment. In such environments students do more than just listen; they are encouraged to explore. When such technology is used in classroom, the focus is on developing higher order thinking skills where students are actively engaged in analysing and synthesising their own knowledge. However, a teacher has to design appropriate classroom activities skilfully, using these

technological tools to enhance mathematics teaching and improve conceptual development and visualisation.

Goldin (1990) states that in a constructivist classroom, learning involves constructive processes. A constructive process is one in which an individual organises and restructures his/her own experience through their own construction of knowledge. The constructivist approach views that learners have a certain amount of knowledge already with them – they do not come to class as empty vessels to be filled. The transfer of knowledge is therefore not one of filling empty vessels, but one of individual knowledge construction.

The hallmark of constructivist teaching is that activities are thoughtfully chosen by the teacher who also allows learners to construct their own knowledge. In a constructivist world where a learner engages with a variety of information and tools to guide his / her inquiry-based learning, Rickards (2003) argues that technology based teaching and learning is complementary as it allows the learners to build on their own existing knowledge. Thus the emphasis is on the teacher constructing a technology based learning environment, and not only on teaching with technology. From a constructivist perspective, Trinidad (2003) found evidence that technological environments can bring about pedagogical changes that involve practical application of new materials and methods. There is a change in the belief of teachers and realisation that learners are no longer dependent only on the teacher for information. Rickards (2003) advocates that utilising discovery based learning functionality made available in a technology based learning environment complements effective teaching. This study advocates that through using GeoGebra for example, selected teachers may foster such an environment.

Sutherland (2007) states that many educational technologies such as GeoGebra have been developed for learning mathematics as it enables students to develop 'symbolic tools' for solving mathematical problems. Such tools become 'internalised' to the extent that they can be drawn upon later in other problem solving situations. The process of internalisation is central to constructivism (Marti, 1996, p. 60). Internalisation is the development of knowledge involving simultaneous re-organisation of internal mental spaces and external forms of knowledge.

Hoyles (Hoyles, 2005) also observes that virtual manipulatives are mediators and form a crucial part of internalising knowledge. Mathematical meanings are inextricably interwoven with the computer tools which enable learners to identify mathematical invariants and make connections. Hoyles (Hoyles, 2005) also argues that the design of tasks using computer tools play an important role in teaching and learning of mathematics that provide an opportunity to build mathematical ideas. Such tools must be able to illuminate properties and relationships of mathematical ideas. In this research study GeoGebra applets designed as part of GLIP will be used by the teachers in the classrooms. Such applets would encourage learners' engagement with mathematical ideas.

6 Research Methodology

6.1 Orientation of the study

Merriam (2009) describes the purposes of qualitative research "to achieve an understanding of how people make sense out of their lives". In this study I am interested in how teachers interpret their experiences using GeoGebra applets in routine teaching and learning of mathematics. Through

observing and discussions with the participating teachers, I hope to understand how GeoGebra applets are integrated in their classroom practice and how the visualisation capabilities of the GeoGebra software are emphasised in building mathematical ideas. Thus a qualitative research approach is apt for my study as it allows me to understand teachers' practices in a technology based classroom.

Scott & Usher (2009, p. 25) write that in the interpretive paradigm, research takes everyday experience and ordinary life as its subject-matter and asks how meaning is constructed. Human action is inseparable from meaning and experiences are classified and ordered through interpretative frames. Merriam (2009) argues that "interpretive research, assumes that reality is constructed, that is, there is no single, observable reality. Rather, there are multiple realities, or interpretations, of a single event." This is significant to my study as I wish to capture and portray all the complexities and facets of the experiences of the participant teachers as they use GeoGebra applets in their teaching.

This study strives for an in-depth understanding of a teaching situation through the eyes of the participants, to get inside the person and understand him/her from within (Cohen, Manion, & Morrison, 2007, p. 21). Thus the key concern here is understanding from the participating teachers' perspective. By being with the teachers for a period of one academic year, I seek to understand the experience of four teachers integrating GeoGebra as a visualisation tool into teaching. The interpretive paradigm will enable a rich understanding of situations (classroom and planning) where teachers utilise their teaching and technological skills and make decisions accordingly. Because this research is being oriented within this paradigm, the events and findings are unique and largely non-generalizable (Cohen, Manion, & Morrison, 2007).

6.2 Methods

This research has elements of micro ethnography and action research methods.

6.2.1 Microethnography

Schwandt (2007) defines microethnography as a qualitative approach "specifically concerned with fine-grained examination of a specific activity within an organisational unit". A microethnographic study, Garcez (1997) argues, examines the processes of how participants create context and make sense during their activities in educational environments. Microethnography involves active engagement of teachers and learners, thus making sense of their interactions in their classrooms, aligning well with active learning in constructivism. GLIP enables teachers to design teaching and learning activities that ultimately provides learners an opportunity to be actively engaged in their learning (Trinidad, 2003). Teachers interact with their applets and make use of these applets to, in turn interact with the students, so that teaching and learning of mathematics takes place. Recognizing that individuals are not passive, but active in making meaning, microethnography explores the interpretative work that individuals engage with, concentrating on 'how' interactions are performed. This microethnographic research tries to understand how teachers interact, respond and 'make sense' (Patton, 2002, p. 110) of their teaching using GeoGebra. As I have a special interest in observing, through constructivist lenses, naturally occurring classroom settings and teachers making sense of their teaching by using dynamic geometry, a microethnography will be an appropriate method of answering my research questions.

6.2.2 Action Research

An important objective of GLIP is to improve teacher practice by incorporating DGS in the mathematics teaching of GLIP teachers. Scott & Usher (2003, p. 36) describe action research as a “systematic study of attempts to change and improve educational practice by groups of participants by means of their own practical actions and by means of their own reflections upon the effects of their own actions”. According to O’Leary (2004) action research is ‘highly participative and collaborative type of research’ which makes it ideal for this research. A main feature of action research is collaboration among the research participants where the distinction between the researcher and researched is minimised while seeking solutions to improve professional practice. Mertler (2012) also argues that action research allows teachers to study and reflect their own instructional methods in order to improve their quality of effective teaching. Accordingly, action research is largely about critically examining what teachers are doing, why they are doing it, and effects of their actions. Educational action research has the potential of engaging and empowering teachers in the process of teaching (Mertler, 2012). In the GLIP project, a group of teachers have come together with a goal of changing and empowering themselves by using GeoGebra in teaching mathematics. The empirical field of this study is thus the GLIP programme, which occurs in cycles defined by the applets that the teachers develop. The action research cycles are thus determined by when the participating teachers implement the applets in their teaching.

Figure 3 below is a diagrammatic representation of the elements of action research adapted for this study as it overlaps with some of the GLIP cycles. For the purpose of this study I will refer to GLIP **cycles** and Action Research **steps**. The first step of the Action Research process is the planning step. Here the participating teachers we will discuss the pedagogical strategies to be used for implementing the designed applets in the classroom. The second step, Act, is the implementation of the developed applets in the classrooms for teaching and learning. In this step I will gather important data as discussed below. The third step is the reflective step where the teachers share their experiences of using applets in the classrooms. The fourth step is an individual interview with the teachers using stimulated recall techniques as discussed below. This is also important data for my analysis.

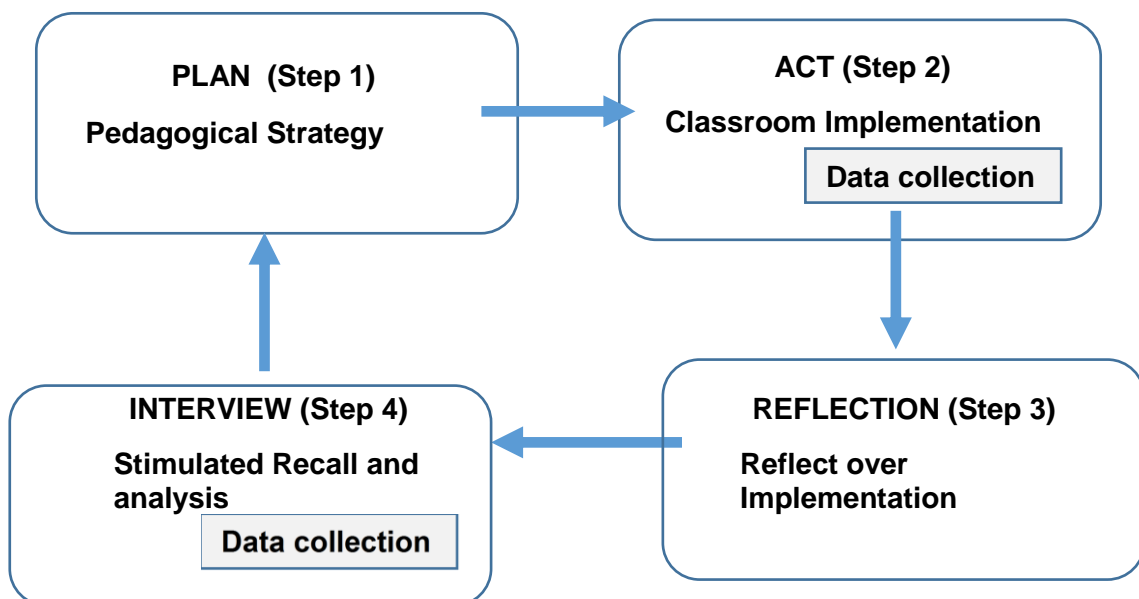


Figure 3: Sketch of the of Action Research steps

Numerous researchers (Gay, Mills, & Airasian, 2012; Mertler, 2012; Scott & Usher, 2003) have developed models of action research processes. Generally, this collaborative process follows a spiral of planning, acting, observing and reflection. The GLIP cycles and its components developed for this study are detailed in the Data Collection section below.

6.3 Participant selection

For this research study, four Grade 11 mathematics teachers from an independent school in Mthatha district in Eastern Cape Province involved in the GLIP will be selected to participate in this research project. These four teachers, my co-researcher and I constitute the six Grade 11 mathematics teachers at the school. These four teachers have in principle already agreed to volunteer for the research as of now. However, their formal approval will be obtained before the actual data collection begins. Anticipating that one of the teachers may opt out of the research participation, if not from GLIP, the research study may focus on three teachers. These teachers are proficient in using DGS. The school computer lab with its interactive board will be utilised by the participants for their teaching. Prior to the data collection, I will design an induction program consisting of 2 sessions with the participating teachers. The purpose here is to introduce the research project, discuss the role and importance of visualisation in teaching and learning of mathematics and confirm their participation. I will also brief them on Kilpatrick's analytical framework for teaching proficiency.

6.4 Data collection

Data collection for this research project begins in the second phase of GLIP where the participating teachers use GeoGebra applets as a pedagogical tool. GLIP is an iterative process of designing, implementing, reflecting and refining applets involving many cycles spread across grades from 9 to 12. For my research, I will use six GLIP cycles with the participating teachers of Grade 11. Each GLIP cycle being driven by one particular applet. These six cycles will be my empirical field of research study. Each GLIP cycle has six components:

- 1) **Selection of mathematical domain:** Here the participating teachers and I will select a mathematical domain and topic on the basis of the Grade 11 annual teaching plan and what specifically the participating teachers would be teaching. We would brainstorm and workshop how we could harness aspects of GeoGebra into the design of our lessons in order to make connections and develop conceptual understanding. Appropriate problems are to be carefully selected as 'not all problems are best investigated with GeoGebra' (Bu et al., 2012).
- 2) **Pedagogical imperative:** After selecting a particular topic, we will discuss some of the possible pedagogical practices in the context of that topic we could implement, and more specifically identify how multiple representations and GeoGebra visualisation processes could be used to enhance conceptual understanding.
- 3) **Design of the applets and planning of lesson:** The applet can be either developed by the teachers or adapted from the GeoGebra website. Two teachers will take the initiative of designing a GeoGebra applet that supports the topic chosen above. All teachers will then assess the applet together and debate about the effectiveness of the applets. We will also discuss the possible teaching strategies using this applet. After the discussions the four teachers will then plan their lesson to implement the applet.

- 4) **Implementation:** Each teacher will then teach his or her lesson with the applets. For each mathematical domain, I will observe two of the four classes. These classes will be chosen in turns so that each participant teacher will be observed in at least three lessons in the six cycles. I will thus observe 12 lessons in total. During the classroom observation I will take the role of a non-participant observer and will audio-video record these lessons. The focus of the observation will be two-fold: firstly, to analyse how the teacher used the applet as a visualisation tool and secondly, how the use of the applet as an integral component of the lesson, enhances teaching proficiency. See my data analysis section below for further details.
- 5) **Reflection and interview:** I will interview the teachers individually after each of the classroom observations, in the form of 'stimulated recall' interview which is further described in data analysis. The purpose of the interview is to gather information and clarification about the teachers' experiences of the lesson using the applet with respect to how the use of the applets facilitate conceptual and instructional fluency. It will also provide an opportunity for the teachers to reflect on the success of their lessons and provide critical insights into the efficacy of the applets as a visualisation tool. The interview will allow me to explore and analyse the teachers' interaction and engagement with the applets against the first two strands of Kilpatrick et al.'s proficient teaching viz. conceptual understanding and instructional fluency. During this interaction I will focus on questions such as: What do you think about this applet? What else could have been done? How did (or did not) GeoGebra help you in expressing visual features of the topic? This participative reflective session will also be audio/video recorded.
- 6) **Refinement:** After each cycle I will consolidate the data obtained in that cycle. The data will inform me as to how far the teachers were able to create a learning environment using GeoGebra applets. Whether the teachers were able to implement the intended pedagogical practices? This component will provide me with an opportunity to review and consolidate what has been achieved in that particular cycle. It will then help me and the teachers to plan and refine the next cycle.

The diagrammatic representation of data collection is as given in Figure 4 below. The mentioned components in each GLIP cycle are only indicative and may change during the process. The adjacent Table 2 indicates the components of the GLIP cycles where the action research takes place.

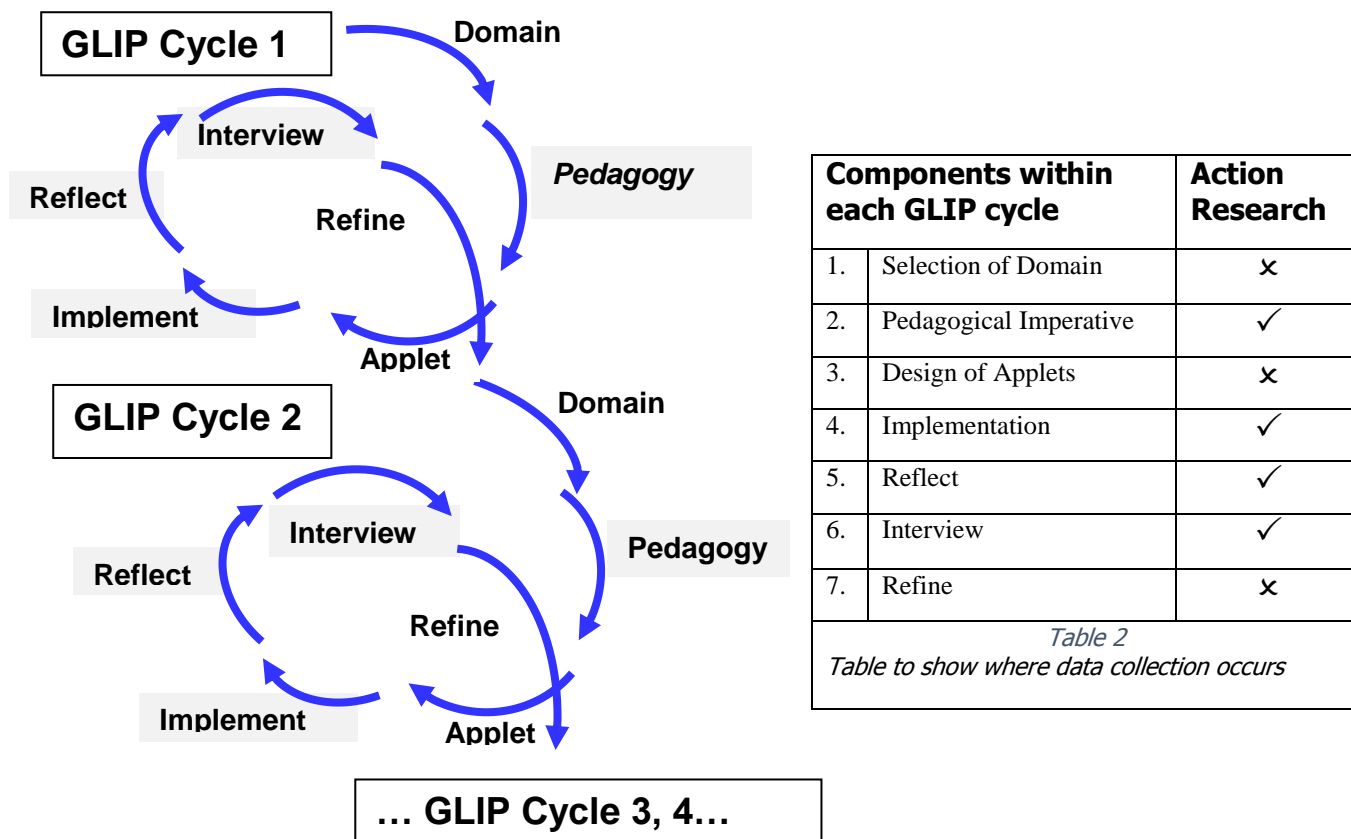


Figure 4 : Cycles of GLIP, the empirical field

6.4.1 Pilot Study

Cohen et al. (Cohen et al., 2007) assert that a pilot study ensures that "the observational categories themselves are appropriate, exhaustive, discrete, unambiguous and effectively operationalize the purposes of the research". I will jointly conduct a pilot study with my fellow researcher for two GLIP cycles. This will help me to test the logistics of the data collection procedure, like audio video recording and transcriptions of these recordings. The pilot data collected from these two lessons will be reviewed and analysed against the instruments CUIF and VATP to check inaccuracies and discrepancies. The review of my own interview questions will expose any ambiguities and inconsistencies. I will then make necessary modifications to the process and research instruments.

6.5 Data analysis

'Data analysis involves organizing, accounting for, and explaining the data; in short, making sense of data in terms of participants' (Cohen et al., 2007). Data analysis will be done parallel to the data collection process i.e. I will analyse the data after each action research cycle. I believe this parallel analysis brings greater focus and the analysis becomes more intense as the study progresses. The process of data collection and analysis is recursive and dynamic (Merriam, 2009).

I will be using two analytical instruments, refer to the Appendix. Instrument 1 is the CUIF (Core Understanding Instructional Fluency) framework adapted from Kilpatrick et al.'s (2001) and Stephanus (2013). When a teacher is teaching new mathematical concepts by bringing in connections between prior knowledge and real life situations, such action will be categorised as Core Understanding for

Connections (CC). Whereas, while interacting with a learner lacking basic skills, if a teacher switches flexibly to another teaching approach, such actions will be classified as Instructional Fluency for acquisition (FA). Instrument 2 is the VATP (Visualisation-Applet-Teaching Proficiency) analytical instrument. It has been developed, based on literature, for linking Kilpatrick et al.'s indicators to visualisation strategies and GeoGebra applets. The VATP is a Likert-type scale to analyse the effectiveness of GeoGebra applets by the selected teachers in their classrooms against the teaching activities. A Likert-type scale can be used to examine quality, frequency of occurrence, or level of comfort (Mertler, 2012). The design of both frameworks are a work in progress and will be refined during the pilot study.

Data from recorded sessions

Firstly, the data collected from the participative sessions with teachers in the form of transcribed audio-video recordings of the lessons will be reviewed a number of times. Data generated from the audio/video recording will be transcribed and documented for analysing against the developed instruments namely CUIF and VATP.

Data from interview

Secondly, after my initial video analysis of each classroom observation, I will ask the participating teachers to reflect on the lesson. This reflective step will be both a data gathering and data analysis session. 'Stimulated recall' interviews can provide the setting to facilitate the reflective process (O'Brien, 1993). In this reflective session, a stimulated recall interview technique will be used where the teachers and I will view the videos of their teaching and interpret the lesson. This method of stimulated recall will prompt teachers to recollect and interpret the strategies they used in the class. The teachers will interpret and analyse their lessons against the CUIF instrument. These videos will be reviewed against the indicators of the two strands (core knowledge in practice of teaching, fluency in instructional routine) of Kilpatrick et al.'s (2001) analytical framework of teaching proficiency. The data will also be analysed on how the teachers use visual strategies in their instructional routines and classroom activities. This analysis will help me to interpret the participant teachers' suggestions, preference and thinking in using DGS in mathematics classroom.

6.6 Validity

Cohen(Cohen et al., 2007) states that in qualitative research validity should be addressed through the honesty, depth, richness and scope of the data achieved, the extent of triangulation and the disinterestedness or objectivity of the researcher. Therefore, by ensuring multiple sources of evidence during data collection, viz, audio-video recording, reflections of teachers, may help to enhance validity of data. Everyday experience and interactions with teachers as participants for a prolonged period of one academic year will provide sufficient scope, rich data and depth to observation.

Triangulation is a primary way to ensure the trustworthiness of data by using multiple methods, data collection strategies, and data sources to obtain a more complete picture of what is being studied and to cross-check information(Gay et al., 2012). Methodological triangulation will be ensured by collecting data using more than one method such as, audio/ video recording during class room observation and open ended interview after implementation (step 4 of each cycle) of applet. According to Maxwell

(Maxwell, 2009) respondent validation is the most important way of ruling out the possibility of misinterpreting the meaning of what participants say and do and the perspective they have on what is going on. Patton (Patton, 2002) defines validation as a procedure aimed at reducing distortions introduced by the inquirer's predisposition. I will adopt a strategy of respondent validation for ensuring internal validity or credibility. The process of respondent validation involves discussion of preliminary analysis and interpretations with the teacher participants before making conclusions (Merriam, 2009). This would be an important way of identifying my own biases and misunderstandings of what I have observed. I will involve all the four teachers in a collective and reflexive mode in getting their views and perceptions on the observed lesson and check consistency of my observations and interpretations.

Validity of Applets: Prior to the development of the applets, the teachers and researchers will brainstorm the design of the applets, taking into consideration of the pedagogical values that are to be embedded in the applets. Once the applets have been developed, the teachers would come together to pilot them amongst each other, analyse and debate their effectiveness. These applets may be improved upon before the actual implementation in the classrooms. Since these applets are thoroughly discussed, debated, analysed and improved by a group of qualified and experienced teachers, they should be valid and hold good for this study.

Validity of Instruments: The assessment instruments developed for the study, the CUIF and VATP, have been adapted from recent literature. The CUIF has been modified from the empirical work of Stephanus (Stephanus, 2013) and Kilpatrick et al. (Kilpatrick et al., 2001). VATP is an adaptation of the classical Likert-type scale to the research context. The pilot study prior to the empirical research will help to validate these instruments.

RU FACULTY OF EDUCATION: ETHICAL APPROVAL APPLICATION

IMPORTANT: The following form needs to be completed by the researcher and submitted with their research proposal to the Education Higher Degrees Committee. The details to which this form relates should also be evident in the text of the proposal.

GENERAL PARTICULARS

MEd (Half thesis)		MEd (Full thesis)		PhD	.	Other: Please specify	
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TITLE OF RESEARCH: An analysis of how the potential of Dynamic Geometry Software as a visualisation tool may enhance the teaching of Mathematics.

DEPARTMENT/INSTITUTE: **Mathematics Education**

DATE: [Submission to EHDC]

RESEARCHER: **Beena Deepak Mavani**

SUPERVISOR/S: **Prof Marc Schafer**

ETHICS

NB: You must read the Faculty of Education Ethics Guideline *prior* to completing this form. Please indicate below how your research supports the indicated ethical principle:

Respect and dignity

The participation of teachers in this study is voluntary and the participants are free to disengage themselves from the study if they wish to do so at any stage of the study. Confidentiality of information collected will be ensured and no part of the collected data will be used for any other purpose other than this study. Participants' identity and school remain anonymous by using appropriate pseudonyms (Gay et al., 2012). The participants will also sign a form that would give their consent about the non-disclosures of the other teachers' practises and view-points. The audio-visual media, the reflective interview transcripts and / or any other data collected will be kept in a safe and secure in a place accessed only by the researcher and her supervisor.

Transparency and honesty

Cohen, Manion, & Morrison (Cohen et al., 2007) argues that participation in the research project should be on the informed consent and voluntary. Informed consent implies that the participants, the four teachers in this context, are voluntarily involved, have a full understanding of the commitment and activities involved. The teachers will also be aware that their classes will be audio-video recorded, through informed consent form and the induction program. These multimedia recordings of the classes will be used to review their lessons along with the other participating teachers including the researcher. The learners' informed consent will be taken care of by my fellow researcher.

Accountability and responsibility

According to O'Leary (2004) accountability expects the research study to be open and transparent. Respecting truth and accuracy, I will be conscious of my obligations to represent the true and accurate perspectives and interpretations of my teacher participants. Accountability in qualitative research implies that through detailed descriptions of the research events and the viewpoints of the participants, any reader may be able to determine the credibility of the research process. As a responsible researcher, I will acknowledge all sources of information.

I am one of the teachers who initiated the GLIP program, but after training of teachers I assumed the role of a participant teacher. During discussions and meetings, I participate like other teachers. However, during classroom observation, I will take the position of a non-participant observer.

Integrity and academic professionalism

To maintain integrity and professionalism in the study, I will provide accurate research accounts of the participants. I will check my interpretation of the data with the participants. The bottom lines of an ethical research are my own integrity and participants' welfare and dignity (Gay et al., 2012).



Signature (researcher)
Beena Deepak Mavani

Date: 27-06-2016



Signature (supervisor)
Prof. Marc Schäfer

Place: Grahamstown

Proposed Research Schedule

	Nov-15	Jan-16	Mar-16	Apr-16	May-16	Jun-16	Jul-16	Aug-16	Sep-16	Oct-16	Nov-16
Teacher Training	█	█									
Workshop Learners				█	█						
Research proposal Approval							█	█			
Applets Preparation					█	█	█	█	█		
Pilot						█					
Ethical Clearance							█	█			
Data Collection											
Data Collection								█	█	█	
1st Cycle								█	█	█	
2nd Cycle									█		
Review / Reflect / Analyse								█	█	█	
Applets Preparation										█	█
	Dec-16	Jan-17	Feb-17	Mar-17	Apr-17	May-17	Jun-17	Jul-17	Aug-17	Sep-17	Oct-17
Thesis Writing											
Applets Preparation			█	█	█	█					
3rd cycle			█								
4th cycle				█							
5th cycle					█						
6th cycle						█					
Review / Reflect / Analyse			█	█	█	█	█				
Chapter 1 Introduction & Background	█										
Chapter 2 Theoretical Prespective		█	█								
Chapter 3 Research Methods				█	█	█					
Chapter 4 Data Analysis						█	█	█			
Chapter 5 Discussions								█	█	█	█

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Appendix
CUIF (Core understanding and Instructional Fluency) indicators based on Kilpatrick et al.'s Teaching Proficiency

<p>The TWO strands of Kilpatrick et al.'s (2001) teaching proficiency</p>	<p>Indicators of the TWO strands of teaching proficiency</p>
<p>Conceptual Understanding (CU) - refers to knowledge of mathematics, knowledge of students and knowledge of instructional practices.</p>	<p>CK: (Knowledge) demonstrates accurate explanations of mathematical concepts, operations, relations and notations that are useful to learners</p> <p>CMR: (Multiple representation) represents mathematical ideas in multiple ways</p> <p>CC: (Connections) emphasizes relationships among concepts and make connections; makes links to prior learning; makes references to real life situations</p> <p>CD: (Discovery) encourages and engages learners in tasks to explore and discover concepts</p> <p>CM: (Mis-conceptions) have identified common student misconceptions and is prepared to remove such misconceptions Non-example: Explaining Mnemonic techniques is not a core knowledge of a teacher.</p>
<p>Instructional Fluency (IF) is ability to use knowledge flexibly in practice to appraise and adapt instructional materials (pg. 369)</p>	<p>FA: (Acquisition) a range of instructional materials which can be drawn fluently as they interact with students</p> <p>FM: (Mis-conception) responds to a learner with serious misconceptions</p> <p>FL: (Literacy) able to deal with students who lack basic skills</p> <p>FR: (Responsive) flexible and responsive to students queries and provide alternate methods or procedures to the same problem</p> <p>FP: (Procedure) explains and encourages learners to use and select appropriate algorithms, formulae, procedures or conventions accurately, efficiently and flexibly</p>

VATP (Visualisation - Applets - Teaching Proficiency) Analysis in Likert- Type Scale

Teaching Activities	Indicator *	Frequency of use of GeoGebra Applets				
		Never	Rarely	Sometimes	Often	Always
		1	2	3	4	5
Interactions with applets:						
Teacher demonstrates concepts						
Teacher brings in multiple representations of the concept						
Teacher brings in connection(s) with other topics						
Teacher encourages discovery of ideas						
Teacher helps to identify common misconceptions						
Teacher promotes switching between abstract and concrete						
Interactions with students (using or without using applets)						
Teacher employs a range of instructions						
Teacher attends to individual misconceptions						
Teacher respond to queries						

*Indicators: Denote 'V' for visualisation; Indicators of Teaching Proficiency mentioned in bold in the above CUIF table.